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Report on the first European Mechanics Colloquium, on the Coanda effect

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The first in a new series of European research conferences in mechanics was held in Berlin on 5 and 6 April 1965. The subject was the Coanda effect, or boundary layers and jets on highly curved walls. Participation was restricted to about 40 people, invited on the basis of their active interest in the subject. These Colloquia are intended to have an informal, workshop-like, character, and formal papers are not normally available, nor will any full proceedings be published. The first author was the chairman of the Colloquium. The following account of the scientific developments of the Colloquium has been prepared to make them widely available. The references quoted give further details of the work discussed at the Colloquium and of related previous work.

1. Introduction

What is now known under the name of 'Coanda' effect† is that rather spectacular phenomenon which one can readily observe by holding a finger against a jet of water running out of a tap: the jet will adhere to some extent to the finger and be deflected (figure 1). Similar observations were made by Young (1800): 'The lateral pressure which urges the flame of a candle towards the stream of air from a blowpipe is probably exactly similar to that pressure which eases the inflexion of a current of air near an obstacle. Mark the dimple which a slender stream of air makes on the surface of water. Bring a convex body into contact with the side of the stream and the place of the dimple will immediately show the current is deflected towards the body; and if the body be at liberty to move in every direction it will be urged towards the current.'

These remarks suggest that the effect to be discussed is part of a more general phenomenon in fluid motions, concerned with a body immersed in a non-uniform stream. A typical example is sketched in figure 2, where the body has the shape of a thick aerofoil with a sharp trailing edge so that separation is fixed there. A jet passes above the aerofoil and it is known‡ that a lift force is exerted on the aerofoil, directed towards the region with higher velocity or total

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† Although this effect has been described in a French patent by Coanda (1932), it has also been observed and explained by others; see, for example, Reynolds (1870), Ackeret (1926), Lafay (1929), Bouasse (1931), Metral (1939), Squire (1945) and other papers listed at the end of this paper. This list also includes references to work not discussed in detail at the Colloquium. See especially the summary paper by Newman (1961).

‡ See, for example, von Kármán (1929), Glauert (1934), Ruden (1939), Küchemann & Weber (1953, chap. 10), Thwaites (1960, chap. 12).

pressure. This is brought about by two main contributory effects. The first is related to the curvature forced upon the jet by the convex body (and also by the circulation round the body, if there is one in the absence of the jet), which is associated with a pressure field involving increased suction forces on the side of the body facing the jet. This effect can be satisfactorily explained on classical potential theory. The second effect is related to the entrainment of fluid into the jet due to turbulent mixing; this induced flow, directed towards the jet, leads to a force on the body in the same direction. This effect thus requires the consideration of viscous phenomena for its explanation. It is useful to remember that both these effects will generally be involved in the particular cases discussed below.

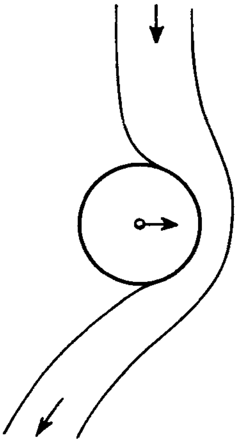

FIGURE 1. Deflexion of a vertical jet by a round body.

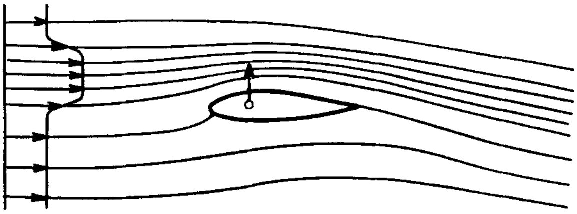

FIGURE 2. Interaction between an aerofoil and a non-uniform stream.

The discussions at the Colloquium were centred mainly on a more specific flow, sketched in figure 3, with the particular features that the jet is adjacent to a curved wall and that the separation line is not fixed, and in which the practical interest is directed towards achieving a large turning angle of the jet. The potential-flow part of the problem is then concerned with a jet which has a free boundary on one side and a solid wall on the other. In the absence of an external stream, the pressure must be constant along the free boundary; it must be lower than this value along the wall. This problem has no unique solution, even if it is assumed that the jet separates somewhere from the wall. Since separation implies that the pressure at the separation line and also downstream of it along the inner free surface of the jet must be equal to the undisturbed pressure again, the pressure distribution along the wall must have an adverse gradient towards the separation line. The viscous-flow part of the problem is, therefore, concerned not only with entrainment effects but also with the development of the boundary layer along the wall, since the latter must be expected to determine the position of the separation line and hence select the particular potential-flow solution which applies. These 'boundary layers' may differ essentially in some respects from the classical boundary layer along a plane wall, as can be seen immediately from the sketches in figure 4. Whereas a jet which is thick in relation to the radius of curvature of the wall may initially develop a conventional boundary layer underneath and a free sheer layer above it (cases (*a*) and (*c*)), a thinner jet will possess profiles of a different nature (cases (*b*) and (*d*)). Only in cases (*a*) and

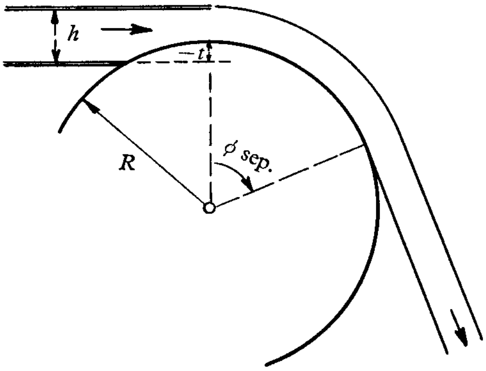

FIGURE 3. Nozzle with jet flow over curved wall.

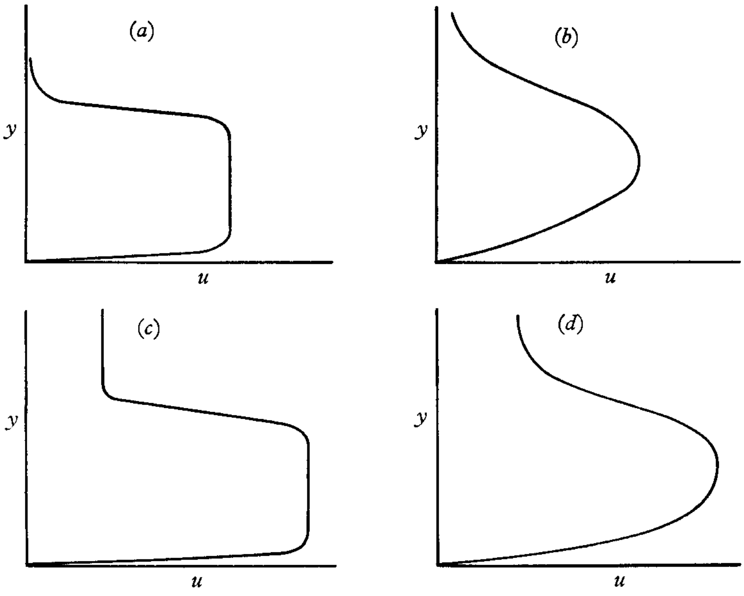

FIGURE 4. Possible types of velocity profile.

(*c*) can one expect that the pressure field is essentially that obtained in a potential flow and that boundary-layer effects determine which potential-flow solution applies. By contrast, cases (*b*) and (*d*) represent flows which are essentially rotational, and the pressure field even in an inviscid rotational curved flow will differ from that in a corresponding irrotational curved flow. This can be expected to have far-reaching consequences and explains why existing theoretical methods are not yet powerful enough for practical purposes and why experimental data must be relied upon to a considerable extent. Matters may be further complicated by the presence of a boundary layer on the outside of the jet nozzle (leading to a local wake-like dip in the velocity profiles (*c*) and (*d*)) and by the presence of step or gap between the jet and the wall (possibly leading to a local bubble-like modification of the profiles shown). Further, even in cases (*a*) and (*c*), possible effects of the curvature of the wall may invalidate the assumption, usually made in boundary-layer theory, that the pressure remains constant across the boundary layer.

This is a brief outline of the main effects involved in the flows under discussion and of the principal problems to be solved. At the Colloquium, an introductory lecture was given by Gersten* who surveyed the physical effects that occur and what is known about them. Below, a brief report is given of the discussion of these problems at the Colloquium, which covered most but not all of the problems listed above. It will be seen that there is still a long way to go before a reasonably complete treatment of these flows will have been established. Available theories, in particular, deal mostly with one or two of the aspects in isolation. The difficulties encountered are much the same as those encountered in other fields of fluid mechanics, viz. viscous flows, involving separation, where classical boundary-layer concepts are inadequate, and turbulent motions.

2. Inviscid wall-jet

Solutions for potential flows rather like that in figure 1 are known, notably that given by Lighthill (1945) and Woods (1954). For each given value of the deflexion angle of the jet, a particular shape of the curved wall is obtained together with the positions of the attachment and separation lines. Lighthill argues that these are realistic and that it is unlikely that the adverse pressure gradient upstream of separation could cause a premature separation because the pressure at and near this hypothetical point would be less than the pressure of the air outside and this would suck the jet back again on to the surface as far as the calculated separation point.

This argument is not necessarily conclusive, however, because the pressure at some other separation point in a real flow could adjust itself to the undisturbed value again. Therefore, Kadosch* attacks the problem in a different way. Using a method developed by Woods (1954, 1961), he calculates flows of this kind, but with the shape of the wall specified to be part of a circle. Pressure distributions along and normal to the wall are obtained, for various assumed turning angles and hence positions of the attachment and separation lines. Subse-

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* An asterisk to a name indicates that the work was described in a lecture at the Colloquium.

quently, the development of a boundary layer in this pressure field is calculated, where the criterion of Gruschwitz and Buri for the separation of an ordinary turbulent boundary layer along a plane wall is used. This then makes the solution unique. The essentials of some of the observed flows can thus be explained but further refinements and extensions with regard to the viscous effects appear to be needed.

Zandbergen* treated a model of the flow as in figure 3. Again, the shape of the wall is specified to be part of a circle and exact potential-flow solutions are obtained. The relatively low pressure in the curved flow downstream of the nozzle is calculated and shown to lead to an increased mass flow from a reservoir at given pressure, as compared with the mass flow in a straight jet. No comparisons with experimental results are as yet available.

It appeared from the discussions that the problem of finding the wall shape for a given pressure distribution, which would seem particularly appropriate to this case, has not yet been tackled.

3. Two-dimensional boundary layers on curved walls

Apart from some first attempts to be discussed later, the effects of wall curvature have been investigated so far only for the case of laminar boundary layers. The usual boundary-layer equations are in effect the first term in an asymptotic expansion of the Navier–Stokes equations for large Reynolds number. Second-order equations can be defined and solutions of these have been considered systematically by van Dyke (1962, 1964) by the method of inner and outer expansions. Such second-order solutions allow the curvature of the wall to be taken into account.

Cooke* reported on his theoretical work which is specifically aimed at assessing the curvature effects. He introduces the simplification originally due to Catherall & Mangler (1963), whereby a 'displacement body' is supposed to be known, but the true body is to be determined from the solution, using the fact that the velocity components vanish on the true body. External vorticity is not considered but could be included in an extension of the method. Similar solutions are obtained, and it turns out that the corresponding first-order solutions are the series calculated by Falkner & Skan and Hartree. Similar but more restricted solutions have been obtained also by Ojha (1965). Earlier calculations by Murphy (1962) are shown to be wrong. Cooke finds that on convex surfaces the skin friction is reduced and the displacement thickness is increased as compared with the first-order solutions. The reverse is the case for concave surfaces. This implies that a convex curvature encourages separation. At the same time, the pressure now varies through the boundary layer and its distribution differs from that obtained in potential flow. However, these effects are not large in incompressible flow for the usual flight Reynolds numbers unless the adverse pressure gradient is very large or the curvature is of the same order as the boundary-layer thickness, when it would be necessary to include further terms in the equations of motion.

This need not be so in compressible boundary layers and especially when high temperatures and large density variations occur. Such a case was investigated experimentally by Stollery* who reported on tests at hypersonic Mach numbers on the flow along a moderately concave wall (actually, a concave corner), where a reduction of the boundary-layer thickness and of the tendency to separation can be expected. This is confirmed, and Stollery finds that the boundary layer remains laminar and survives extraordinarily large adverse pressure gradients without separation. An empirical separation criterion involving both Mach number and Reynolds number is given. These results would appear to imply that these curvature effects may play a more important part when the deflection of high-speed hot jets is considered.

Other second-order effects in boundary layers, involving pressure variations through them, were discussed by Walz.* So far, conditions near the leading edge of a flat plate have been considered.

4. Theoretical investigations of the wall jet in still air

The 'wall jet' in still air is a flow of the type sketched in figure 4(*b*). It can be expected to occur at sufficiently large distances from the exit nozzle of the jet so that the initial boundary layer along the wall and the free shear layer have merged. The first theory of the laminar and the turbulent wall jet, for plane walls, was developed by Glauert (1956), on the basis of a model of the flow involving two regions divided by the line of maximum velocity. In the inner region, the flow is assumed to be dominated by wall effects and to behave rather like a boundary layer. The flow in the outer region is assumed to behave more like a free jet, and a solution is obtained for this region using an eddy viscosity which is constant across this region. Prandtl's boundary-layer equations are used throughout and the solutions in both regions are based on similarity arguments. It is only the joining conditions that give rise to small departures from true similarity.

The theory for the outer region of wall jets is closely related to those for wakes and axisymmetric and plane jets. Tollmien (1926) uses Prandtl's mixing-length concept for the jet and Schlichting (1964) uses it for the wake, whereas Görtler (1942) obtains solutions for both the self-preserving plane jet and wake using a constant eddy-viscosity coefficient. All the velocity profiles obtained are broadly similar even though it is assumed that $\tau \sim \partial u/\partial y$ for the constant-eddy-viscosity solutions and that $\tau \sim (\partial u/\partial y)^2$ for the mixing-length solutions. Glauert's solution has successfully been applied in many cases, even when the wall was curved and when the assumptions made in its derivation did not strictly apply. It was emphasized again in the discussions that the calculated velocity profiles are not very sensitive to the assumptions made for the detailed shear stress distribution.

Glauert's two-layer model forms the basis of an approximate solution by Fernholz* (1965) for a flow as sketched in figure 3. The outer region is regarded as a free shear layer with similar velocity profiles, an assumption which is not strictly justified in the initial development near the nozzle or slot and near the separation line. The inner region is regarded as a turbulent boundary layer with transverse pressure gradient, and the appropriate momentum and energy equations are derived. The initial velocity profile in the nozzle, the distribution of the maximum velocity along the jet, and certain functions relating to the transverse pressure gradient are presumed to be known. The resulting equations can be solved using the methods of Runge–Kutta or of Walz (1958, 1959, 1960). The development of the wall jet, including the position of the separation line, can then be calculated.

Guitton* reported on an extension of his earlier work (Guitton 1964). The boundary-layer equations are solved for the flow along a slightly curved surface of constant radius and the solution is obtained in the form of a power series (in terms of the ratio between boundary-layer thickness, δ, and radius of curvature, R, truncated after the first term). The use of the concept of eddy viscosity is an essential part of this theory which is supplemented by turbulence measurements in a curved flow.

Gersten* divides the flow into an entrance region of four to six slot widths (like that sketched in figure 4(*a*)), followed by a similarity region in which the velocity profile is compounded in the manner of Glauert. This profile is inserted into the equations of motion expanded in powers of δ/R and truncated after the second term. Use is made of some empirical relations proposed by Newman (1961) and by Nakaguchi (1961), and this enables Gersten to define a function $\chi(\phi)$ of an angular coordinate ϕ such that

$$\frac{p_\infty - p_w}{\frac{1}{2}\rho u_m^2}\frac{R}{\frac{1}{2}y_m} = \chi(\phi).$$

As in the work of Newman, this is an attempt to calculate the transverse pressure difference across the jet taking account of the fact that the curved flow is not irrotational. The method still requires the separation line to be specified and the work is continuing.

Riedel* is concerned not with a wall jet on a circular cylinder but with a jet impinging on a curved aerofoil surface in still air. Distinguishing between three regions—the highly curved leading edge, the moderately curved middle part, and the trailing edge—Riedel simplifies the equations of motion according to a concept which allows only for shear forces along the wall and entrainment by turbulent mixing. This involves some order-of-magnitude assumptions which were challenged in the discussion; it was claimed to lead to criteria governing attachment and separation of the jet. In this context, Krämer pointed out that sinks in a potential flow could be used to represent entrainment and referred to Squire's (1950) exact solution of the Navier–Stokes equations for a laminar axisymmetric jet.

5. Theoretical investigations of the wall jet in a moving stream

The flow considered now is of the type sketched in figure 4(*d*). Its essential feature is that the velocity profile does no longer tend to zero at a large distance from the wall but to a finite value, u_0, which is in general smaller than the maximum value reached somewhere in the jet. In practical applications, the external stream may have a temperature different from that in the jet and density variations may also be due to different molecular weights if two different gases are involved. Although mixing processes of this kind have been studied (see e.g. Szablewski 1965), very little is known about them in the context of a wall jet.

The simplest approximation to the spreading of a jet in a moving stream is obtained by superposing the external flow with velocity u_0 on the jet flow into still air. It is then assumed that for the same velocity difference $u-u_0$ the mixing process is also the same. A consistent method involves a stretching of the streamwise co-ordinates, to take account of the different distances which particles travel in unit time in the two cases. This may be regarded as a transformation from a fixed co-ordinate system in still air to a moving one. This method (see Küchemann & Weber 1953, §10–2) gives adequate answers for practical purposes in simple cases. The same method has also been used by Eskinazi & Kruka (1964).

Nicoll* presented an account of Spalding's method (Spalding 1964). This theory rests on two main postulates. The first is that the velocity profiles can be described by relations having two main components, one accounting for the effect of momentum transfer and wall shear, and the other accounting for the interactions with the main stream; thus the general profiles have both what Spalding calls 'boundary-layer' and 'jet' components. These velocity profiles are similar to those put forward by Rotta (1950, 1962), Ross & Robertson (1951), and Coles (1956). The second postulate is that fluid is entrained into the wall layer in the same manner as it is into a turbulent jet and in accordance with similar quantitative laws. This entrainment hypothesis is similar to that put forward by Head (1960); it is now linked to a particular postulate about the velocity profiles. The method is extended to include also the effects of heat and mass transfer. The method is particularly successful in predicting the local drag force and its results are in better agreement with experimental results by Sigalla (1958) and by Bradshaw & Gee (1960) than Glauert's predictions. Spalding's method has another advantage over other methods in that it can be used in situations in which a wall-jet profile changes slowly to a wall boundary-layer profile as it passes downstream.

Harris* reported on an entirely different approach to the problem: He uses the momentum and energy integral equations and assumes that the velocity profiles in both the inner and outer regions are everywhere similar. For the evaluation of the integral $\int\tau(\partial u/\partial y)dy$ in the energy equation, a skin-friction formula similar to the Blasius expression is employed.† Prandtl's mixing-length theory is applied, and the equations are solved by means of a computer. An interesting feature of Harris's theory is that an attempt is made to make some allowance for the fact that the shear stress is not zero at the velocity maximum in wall jets. This has been observed experimentally by Bradshaw & Gee (1960) and by Eskinazi & Kruka (1962) and would imply that theories based on the gradient transport of turbulence cannot strictly apply; it also suggests that the independence of the inner and outer regions is not a good assumption.

So far, Harris has worked out the case of a turbulent wall jet along a plane surface with arbitrary pressure gradients. These solutions are now used to find simple approximations for the numerical results in a form which allows 'influence

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† This assumption was challenged in the discussion.

functions for the external stream' to be defined so that the profiles with external stream can be obtained from those without it by multiplication with these functions. The latter are intended to be of a universal nature so that practical cases can be calculated by hand. The method is to be extended to cover also flows along curved surfaces.

The work of Harris demonstrated the power of modern computer techniques in obtaining numerical solutions. At the same time, it became clear during the discussion how much more complex real flows with various initial conditions and both longitudinal and transverse pressure gradients are than those that can now be treated. Our ignorance about how to take account of turbulent motions was also much deplored.

6. Experimental investigations of jet deflexion by curved walls

The discussion of experimental results was very suitably opened by an ONERA film, made and commented on by Werlé.* The flow in a water tunnel is visualized by various means. The Coanda effect is clearly demonstrated on a model of an air-cushion platform where air is ejected horizontally along a plate and then turned downwards by means of a circular cylinder, sometimes followed by a hinged plane flap. Other cases investigated include blowing over a flap deflected at the trailing edge of a delta wing and blowing over a highly curved convex surface. The flow visualization was of excellent quality.

Bradbury* described his experiments on a simplified blown-flap configuration, comprising a two-dimensional jet ejected over a circular cylinder which has a small flap attached to it. The flow is thus of the type sketched in figure 3. But the flap provides a sharp trailing edge at which separation is intended to occur and the main object of the tests is to find out how far this flap and with it the jet can be deflected before separation occurs upstream of this trailing edge. The jet thickness was half the cylinder radius for the bulk of the tests. There was no mainstream but provision was made for an auxiliary jet of the boundary-layer-control type, emerging from a slot on the cylinder surface. The flap deflexion angle at which separation occurs is measured over a range of jet pressure ratios up to 2·5. Bradbury reported that the flap angle at separation decreased with increasing pressure ratio and that blowing through the auxiliary slot increased the flap angle at separation for pressure ratios below 2 but had almost no effect at higher pressures. A new and more elaborate model on which experiments were just beginning was described together with the results of some measurements in a region where the flow may be regarded as incompressible. These show that the flow is unusually sensitive to changes in Reynolds number when the ratio, R/h, between radius of curvature and jet thickness is smaller than about 4. Similar anomalies have been observed by Koester & Löhr (1964) and Fernholz (1965*b*) and it became clear during the discussion that much remains to be explained before the effects of Reynolds number and of Mach number on these flows are fully understood.

Nicoll* presented some comparisons between experimental results and those from Spalding's theory. As mentioned above, this theory describes the experimental phenomena quite well in most cases. It can also readily accommodate empirical data as they come along; thus a new expression for the entrainment mass flow was presented, which fits some recent experimental data better than Spalding's original relation.

Experimental results for a two-dimensional flow like that in figure 3, but with the cylinder surface not only tangential to the jet but also penetrating into it by a distance t, were discussed by Gersten*. The penetration ratio, t/h, the jet thickness ratio, R/h, and the Reynolds number of the jet were varied. The pressure distribution around the cylinder is found to depend strongly on t/h and also on R/h for the lower range of values of this parameter. The jet deflexion angle also depends on the penetration and on the Reynolds number in accordance with the observations of Bradbury. A value of t/h of about $-0{\cdot}4$ produces the largest deflexion angles. The measured velocity profiles agree closely with Glauert's prediction for the flat wall jet.

Three-dimensional effects produced in such a flow by the presence of side walls were discussed by Fernholz (1965*a*). In his experiments, the cross-sectional aspect ratio, b/h, is varied between 1 and 4 and R/h between 3·8 and 14, at a constant jet pressure ratio $p_0/p_\infty = 1{\cdot}03$. It is found that the detailed shaping of the nozzle exit has an important effect on the deflexion angle achieved, in the range of the parameters tested. Even with strong albeit symmetrical disturbances by the side walls, the velocity profiles near the wall agree closely with Glauert's predictions; the outer parts of the profiles become similar at larger distances from the nozzle. Velocity profiles with two maxima are observed for highly curved and thick jets. Separation may occur if R/h becomes too small or t/h too large; it may take a three-dimensional form of a fanning-out phenomenon in a preferred direction; and it may be associated in the usual way with an adverse pressure gradient far downstream of the nozzle. The law of the wall proved to be valid in the flow near the curved surface.

Lehmann* demonstrated that a small spoiler (of height $= 0{\cdot}03h$), inserted at the outer tip of the nozzle opposite to the curved wall, can greatly increase the deflexion angle and also prevent the fanning-out 'stall' in this three-dimensional flow. Symmetrical insertion tends to make the flow more parallel; a skewed insertion allows the flow to be deflected sideways towards the direction of the generators of the cylinder.

Harris* reported that the results from experiments with a wall jet along a flat surface agreed well with his theory, discussed above. Recent results for a curved surface—a Joukowski aerofoil—also gave good agreement. Detailed experiments are planned (at the von Karman Institute for Fluid Dynamics) on a number of configurations involving plane, parallel and inclined, jets, as well as curved jets, including one over a wall shaped as a logarithmic spiral for comparison with the theory of Sawyer (1963) for this case. Velocity profiles, the turbulent structure, skin friction and pressure distributions along the surface are to be measured.

Riedel* reported on experiments on an RAE 100 aerofoil of 25 % thickness-chord ratio with a jet along one side of it in still air. Separation and reattachment of the jet were observed and pressure and wall shear distributions were measured.

The general discussion of the experimental data and their analysis revealed a fairly optimistic view of current progress although it was realized that a reasonably complete understanding is not yet in sight. Glauert sounded a warning by pointing out that agreement between experimental velocity profiles and his calculated profile, which had been generally noted, did not necessarily imply that the assumptions made about the shear stress distribution had been correct, because quite wide variations in shear stress (due, for example, to curvature) would alter the velocity profile only marginally. Bradbury pointed out that more accurate empirical rules for, say, energy dissipation parameters did not imply a better physical understanding, and there was a general feeling that numerical calculations of turbulent flows from first principles might never even be possible.

7. Various applications

The so-called Coanda effect has a great many practical applications. Most of these lie in the field of aviation and some of the experimental set-ups discussed above directly suggest applications to circulation control of aerofoils and jet-deflexion devices. It has also been claimed that a thrust increase, as well as a change in direction, may be obtained when suitably curved surfaces are placed near a jet. In principle, such a thrust augmentation could be the result of an increased Froude efficiency of the jet, which in turn could be achieved by increasing the mass flow through a mixing process with the air surrounding the jet at a pressure below the atmospheric. The mixing itself could be particularly intense in a Coanda-type flow. The principles of thrust augmentation by mixing have been discussed by Busemann (1939). Evaluations of tests of several research workers by Fernholz (1964) on a jet turned through 90° have shown that the ratio between the vertical component of the thrust and the total thrust without deflexion increases with increasing radius R/h, and is nearly independent of the jet pressure ratio. But the maximum value of this ratio has not exceeded unity in tests by von Glahn (1958), Bailey (1961) and Roderick (1961). Gates & Burdick (1963) reached values of 1·05 and Hope-Gill (1964) achieved 1·19 in a special rig with a surrounding shroud, as in Reid's tests (1962). In all experiments where there has been no shroud, no thrust augmentation has been found. An assessment of the practical value of thrust augmentation can easily be confused by unsuitable definitions of the thrust datum, and the discussion of this point remained inconclusive.

The effect finds another application in the cooling of circular cylinders by jets rather than by a uniform stream of air. Schuh* (1964) reported on his work on the heat transfer from a hot cylinder, where he finds that the heat transfer can be increased by 20 %. The wall temperature was kept uniform and constant. Separation can be delayed to 170° and intensified turbulent mixing also contributes to the increase in heat transfer as compared with that obtained in a uniform stream. In this context, similar experiments by Bhattacharya & Wille (1959) were mentioned; these are relevant to the cooling of moulds in the manufacture of glassware.

The deflexion of a jet along a curved wall has been used to drive a new kind of reaction turbine. This was described by Teodorescu-Tintea* (1960, 1964, 1965). The working fluid, combustion gas or hot air, is discharged through nozzles on a rotating hollow shaft and it is then deflected along adjoining backward-curved 'blades'. The resulting forward reaction force drives the system. In contrast to other reaction turbines, the fluid does not flow in a channel, but only along the convex surface of a blade, leaving the concave surface open for cooling.

Still another application is found in swirl atomisers, as developed by Klein* (1965). Air is blown down the middle of the nozzle and swirling liquid round the rounded lip. With a sharp-edged lip, atomization was found to be poor, but with a curved lip the Coanda effect operated and atomization was very good. The flow of the thin film of liquid is subject to capillary and centrifugal forces, surface tension, and friction between liquid and wall, and only to a small extent to air forces. The separation of the liquid film from the wall is the result of an instability of the interface between liquid and air. Waves appear, with their fronts in the direction of the flow; their amplitude may grow quickly and the film breaks up into separate jets and finally into droplets. Between these jets and the wall striations can be observed; their wavelength depends on the properties of the medium. Klein* reported on experiments with water, solutions of the water and glycerine, methylated spirit, and mercury, carried out in air and in near-vacuum. Preliminary tests were done on a circular cylinder with a nearly two-dimensional flow like that in figure 3, where the instability of the film and its breaking-up could also be observed. In a turbulent flow, the separation phenomena remain basically the same, although they are more complicated in detail. They are also much the same in air and in near-vacuum, as long as the pressure remains above the vaporisation pressure of the liquid. The wavelength of the striations and the turning angle are found to decrease when the mean velocity in the film increased. A definite dependence of the turning angle on the Reynolds number is found only for media of small viscosity.

Two communications to the Colloquium were concerned with the application of the Coanda effect in fluid logic devices. The general principle is based on the facts that a jet discharged into a diverging channel is free to attach itself either to one wall or the other and that it may be forced to separate from one wall and to attach to the other, and hence be controlled by small auxiliary jets emerging from holes in each wall, acting as signals. The system acts as a relay, or amplifier, with hysteresis. The description of the process depends on a suitably defined Reynolds number, and the power needed to operate it (which is roughly proportional to Re^3) leads to values of the Reynolds number which often lie in the region of a critical Reynolds number signifying transition from laminar to turbulent flow. This may have upper and lower bounds because of the hysteresis phenomenon, and the values may strongly depend on the particular geometry used.

Glättli* reported on the use of such fluid amplifiers in computing circuits. The non-linearity of the device is used and needed for binary circuits or for memory circuits. Their response is limited by possible cavitation, various effects depending on Reynolds number and Mach number, and possible contamination by dirt.

The fastest fluid amplifier (developed by I.B.M., Zürich) has a nozzle of 0·05 mm diameter, operates in 5 msec and takes 50 mW. Examples of several circuits were given.

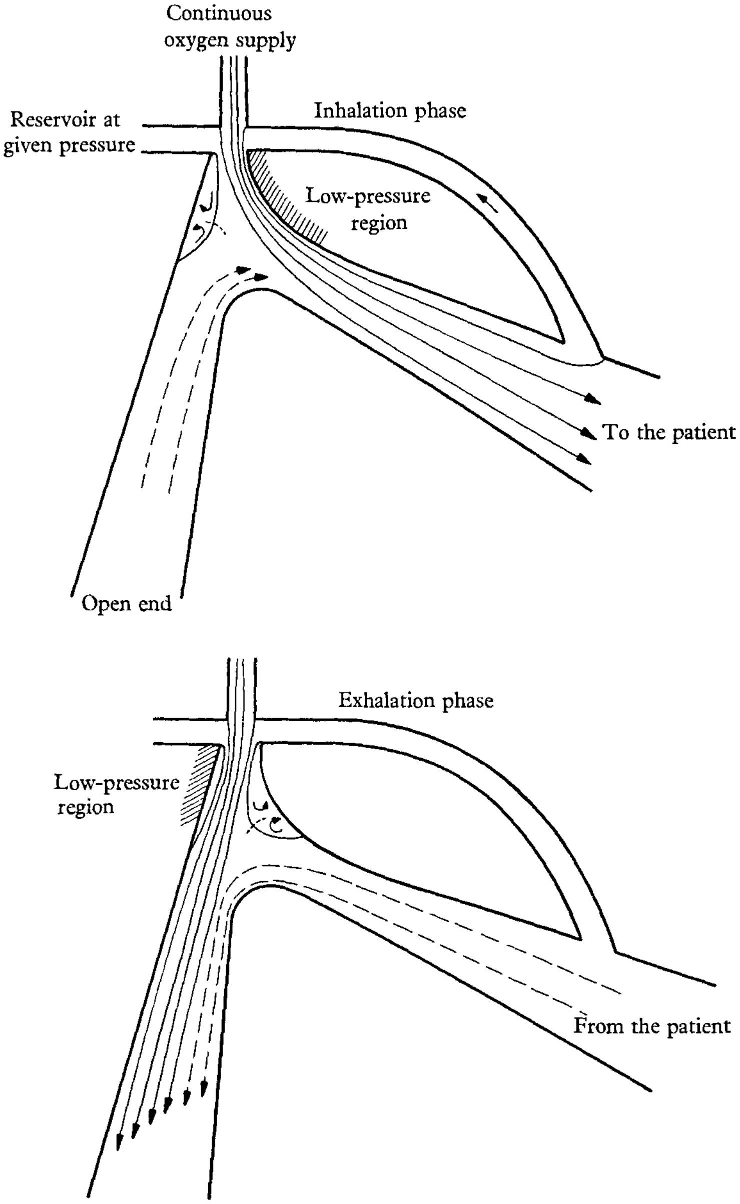

FIGURE 5. Scheme of artificial respirator after Pavlin* (Bertin).

Pavlin* discussed such flows in general terms and referred in particular to subsonic and supersonic flows involving separation and reattachment, which are related to base flows as studied e.g. by Korst (1956) and Carrière & Sirieix (1960). The mixing region demands particular attention, and the work is based on that of Levin & Manion (1962) who use Görtler-type velocity profiles, and on refinements due to Sawyer (1963).

Pavlin also described his experiments on an artificial respirator without moving parts (developed by Bertin and Cie, Plaisir-les-Gatines). Two phases in the operation of this interesting device are sketched in figure 5. In principle, this amplifier is a branched channel, with small tubes for input leading to each side of the throat: As the pressure builds up in the lungs of the patient during the inhalation phase, a point is reached when the mass flow in the small return tube is sufficient to deflect the main stream into the other branch of the device, thus initiating the exhalation phase. The geometry of the channels, with one curved wall and one plane wall with a sharp corner on the other side, provides the necessary bias for a periodic operation. A film showed how well this simple device, based on some fundamental principles of fluid mechanics, works.

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